SOME REMARKS ON THE LINEARIZATION OF THE EQUATIONS OF PLASTICITY

(NEKOTORYE ZAMECHANIIA O LINEARIZATSII URAVNENII PLASTICHNOSTI)

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V. V. SOKOLOVSKII
(Moscow)

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Some remarks are herein made concerning the question of the linearization of the equations of plastic flow as applied to the simple problem of the drawing of a thin tube through a frictionless conical die (Fig.1).

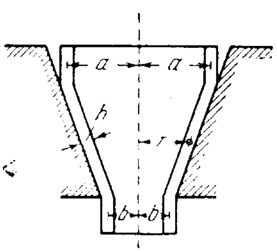

Fig. 1.

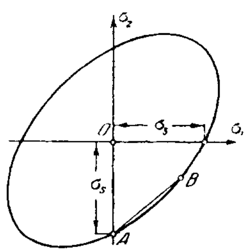

Fig. 2.

We define the stress- and strain-rate fields in the conical tube by the stress components σ_1, σ_2 the rate-of-strain components ϵ_1, ϵ_2 and the radial velocity v, remembering that

$$\varepsilon_1 = \frac{dv}{dr}, \qquad \varepsilon_2 = \frac{v}{r}$$

The differential equation of equilibrium of the conical tube of thickness h has the form

$$\frac{d(h\sigma_1)}{dr} + \frac{h(\sigma_1 - \sigma_2)}{r} = 0 \tag{1}$$

while the plasticity condition is

$$\Phi(\sigma_1, \sigma_2) = \sigma_s \tag{2}$$

The relations between the stress components and the rate-of-strain components are

$$\frac{\varepsilon_1}{\partial\Phi/\partial\sigma_1} = \frac{\varepsilon_2}{\partial\Phi/\partial\sigma_2} \quad \text{or} \quad \frac{\varepsilon_1}{2\sigma_1 - \sigma_2} = \frac{\varepsilon_2}{2\sigma_2 - \sigma_1}$$

which lead to

$$\frac{dv}{dr} + m\frac{v}{r} = 0, \qquad m = -\frac{\partial\Phi/\partial\sigma_1}{\partial\Phi/\partial\sigma_2} \tag{3}$$

The usual condition of incompressibility of the material requires that

$$\frac{d}{dr}(rvh) = 0 \tag{4}$$

We note that even small variations in the function Φ may cause large changes in $\partial\Phi/\partial\sigma_1$ and $\partial\Phi/\partial\sigma_2$, and hence in the coefficient m.

A solution of the above problem obtained by Swift [1] was based on the usual plasticity condition

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_s^2$$

which may be represented by an ellipse in the σ_1, σ_2-plane. The stress components σ_1, σ_2 at the place where the tube enters the die and at the place where it leaves it are depicted by the points A and B (Fig. 2). From this it is clear that

$$\Phi^2 = \sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2, \qquad m = -\frac{2\sigma_1 - \sigma_2}{2\sigma_2 - \sigma_1}$$

The stress components σ_1 and σ_2 in this solution, as well as the radial velocity v and the thickness h, may be represented in closed form, and

$$rvh = av_0h_0$$

We note that for $b/a = 0.425$, the principal stress σ_2 at the point where the tube leaves the die is equal to zero, while the ratio $h/h_0 = 1.054$.

A graph of the function h/h_0, which determines the variation of the thickness of the conical tube along a generator, is shown in Fig. 3.

A solution of the same problem was presented by Prager [2] for a linearized plasticity condition

$$\mu\sigma_1 - \sigma_2 = \sigma_s$$

which can be represented by a straight line in the σ_1, σ_2-plane.

The stress components σ_1, σ_2 at the place where the tube enters the die and the place where it leaves it are denoted by the points A and B,

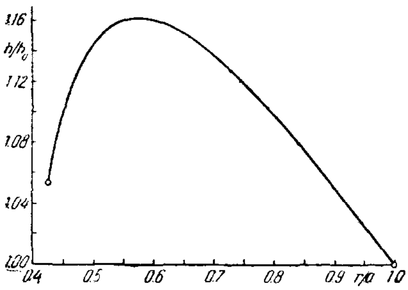

Fig. 3.

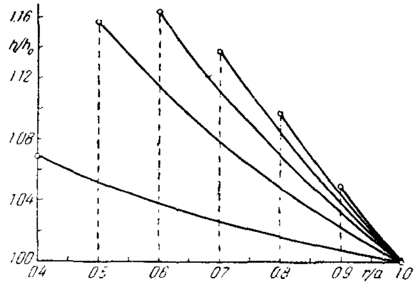

Fig. 4.

as before. It is clear that

$$\Phi = \mu\sigma_1 - \sigma_2, \qquad m = \mu$$

The stress components σ_1 and σ_2 take the form

$$\sigma_1 = \sigma_s \ln\frac{a}{r}, \qquad \sigma_2 = \sigma_s\left(\mu\ln\frac{a}{r} - 1\right) \tag{5}$$

and the radial velocity v and thickness h are

$$v = v_0\left(\frac{a}{r}\right)^{\mu}, \qquad h = h_0\left(\frac{a}{r}\right)^{1-\mu} \tag{6}$$

The parameter μ should be determined from the condition that the stress components σ_1 and σ_2 at $r = b$ satisfy the relation

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_s^2$$

or that the point B lie on an arc of the ellipse.

This condition determines the relation

$$\frac{b}{a} = \exp\left(\frac{1-2\mu}{1-\mu+\mu^2}\right)$$

which yields

$b/a = 0.4$	0.5	0.6	0.7	0.8	0.9	1.0
$\mu = 0.927$	0.789	0.702	0.637	0.584	0.539	0.500

We remark that for a ratio $b/a = 0.368$, i.e. for $\mu = 1$, the stress component σ_2 at the point where the tube leaves the die is equal to zero, while the thickness of the tube is the same everywhere, i.e. $h = h_0$.

Curves of the function h/h_0, representing the variation in the thickness of the conical tube along a generator, are presented for various values of b/a from 0.4 to 1.0 in Fig. 4.

Comparison of the graphs of the functions h/h_0 in Figs. 3 and 4 shows that they have different forms, although the corresponding stress components σ_1 at the place where the tube leaves the die are in fairly close agreement.

BIBLIOGRAPHY

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2. Prager, W., *An Introduction to Plasticity*, Addison-Wesley, 1959.

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